An experimental determination of damping ratio of a multi-storey building subjected to aerodynamic loadings

Onundi L. O.¹*, Elinwa A. U.² and Matawal D. S.²

¹Department of Civil and Water Resources Engineering, University of Maiduguri, Borno State, Nigeria.
²Civil Engineering Programme, Abubakar Tafawa Balewa University, Bauchi, Nigeria.

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The present study focused on an experimental determination of damping ratio of a multi-storey building subjected to aerodynamic loadings. It used the methods of model construction, wind tunnel and dimensional analysis for the investigation. Although, the wind tunnel and dimensional analysis were successfully used in the past and in different ways by many Scholars and Engineers to investigate the problem of wind effects on buildings; but such solutions or efforts, did not include the influence of the building deflection (δ). The results of the study show that depending on the phase of the force with respect to motion, self-excited forces can be associated with the displacement, the velocity or acceleration of the structure. Due to the influence of these associations, these forces can be thought of as “aerodynamic contributions" to stiffness, damping and mass, respectively. The building deflection is therefore, considered to be an important structural quantity which affects structural quantities such as forces, moments, velocity and acceleration which severely affect the human perception criteria of occupants using multi-storey buildings. The damping ratio also proved to have been influenced by dependent variables such as stiffness, amplitudes of vibration, H/B ratio, Strouhal and L₀ numbers at a basic wind speed of 52.5 m/s⁻¹. Therefore, it can also be inferred that the relationship in the variations of the values of H/B ratio and L₀ number are inversely proportional to the values of the Strouhal numbers and damping ratio along the model height. Whereas, the amplitude of the vibrations progressively increased, while, a critically damped model and its stiffness decrease with increase in model height. All these are likely to provide wider areas of applicability in structural analysis and in particular, experimental use of wind tunnel for structural modeling and similitude.

Key words: Dimensional analysis, Strouhal number, Bernoulli universal constant, wind loads, wind tunnels, multi-storey buildings, damping ratio, aerodynamics.

INTRODUCTION

There has been persistent destruction of lives and property caused by direct or indirect effects of wind storms in many areas in the World. Windstorms pose variety of problems to buildings (in particular, tall buildings) and constitute the largest cause of economic and insured losses and Nigeria is not an exception.

According to Taranath (2005), winds that are of interest for the design of building can be classified into three major types: prevailing winds, seasonal winds and local winds. All three types of wind are of equal importance in design. However, for the purpose of evaluating wind loads, the characteristics of the prevailing and seasonal winds are analytically studied together, whereas those of local winds are studied separately. This division is brought about by the scale of fluctuations since the prevailing and seasonal winds speeds fluctuate over a period of several months, but the local winds vary almost every minute. The variations in the speed of prevailing and seasonal winds are referred to as fluctuations in mean velocity; whereas, the variations in the local winds
are referred to as gust.

According to Halvorson and Isyumov (1982) and Taranath (2005), unlike the mean flow of wind which can be considered as static, wind loads associated with gustiness or turbulence change rapidly and even abruptly, creating effects much larger than if the same load were applied gradually. Wind loads therefore, need to be studied as if they were dynamic in nature. The intensity of wind therefore depends on how fast it varies and also on the response of the structure. Therefore, whether the pressures on a building created by a wind gust, which may first increase and then decrease, are considered as dynamic or static depends to a large extent on the dynamic response of the structure to which it is applied. The action of wind gust depends not on how long it takes the gust to reach its maximum intensity and decrease again, but depends on the period of the building itself. If the wind gust reaches its maximum value and vanishes in a time much shorter than the period of the building or structure, it is dynamic. On the other hand, the gust can be considered as static loads, if the wind load increases and vanishes in a time much longer than the period for the building.

When multi-storey steel framed building is subjected to dynamic loadings such as harmonic excitation, it is forced to vibrate at the same frequency of excitation. These excitations may be undesirable or harmful to human perception, the safety of equipment or structure when large vibration amplitudes develop. To prevent large amplitudes from developing, dampers and absorbers are often used (Kareem, 1997; Kareem et al., 1999).

In designing a high-rise building which satisfies the perception and comfort requirement of occupants, the fundamental natural period is a very important design variable. This is very crucial since the design dynamic loading that causes the vibration of the building under the influence of wind induced vibration or wind pulsation is usually influenced by the fundamental natural period and characteristic fundamental natural mode of vibration (Onundi, 2010).

Therefore, to effectively and efficiently design tall buildings against the dynamic effect of wind, considerations such as the site basic wind speed, building dimensions, shape and stiffness, damping ratios, site topography, climatology, foundation type, boundary layer meteorology, bluff body aerodynamic and probability theory are important.

From the recent research experiences, there are many situations where analytical methods cannot be used to estimate certain types of wind loads and associated structural response. For example, when the aerodynamic shape of the building is rather uncommon or the building is very flexible so that its motion affects the aerodynamic forces acting on it. For instance, the building is usually considered flexible when any of the plan dimensions (length or width) divide by height is greater than five or when the minimum frequency is less than unity (Taranath, 2005). In such situations, more accurate estimates of wind effects on buildings can be obtained through aerelastic model testing in a boundary-layer wind tunnel. The oscillator of the structural system is naturally wind forces or its gustiness; but for the purpose of the physical model for this investigation, the wind tunnel served as the oscillator. Therefore, a hybrid effort from the theoretical and laboratory investigations will usually produce the best result.

When a dynamic system is excited by a suddenly applied non-periodic excitation $F(t)$, the response to such excitation is called transient since steady-state oscillations are generally not produced and poses no problem to the structure; if however, the suddenly applied load is periodic excitation $F(t)$, the response to such excitation produces a steady-state oscillations which may be harmful to the structural system. Therefore, the objective of this work is to experimentally measure the degree of aerelastic damping caused by the effects of dynamic wind pulsation on a multi-storey building due to the influence of steady state oscillations. Depending on the phase of the force with respect to motion, self-excited forces can be associated with the displacement, the velocity or acceleration of the structure. Due to the influence of these associations, these forces can be thought of as “aerodynamic contributions” to stiffness, damping and mass respectively (Mario, 1991).

Therefore, the aim of this investigation is to experimentally measure the associated deflection at the top (top drift) of a fabricated and tested physical model in an Eiffel-Type subsonic boundary layer wind tunnel, all the relevant parameters that affect the dynamism of multi-storey building will be determined through the use of method dimensional analysis and other relevant analytical structural approaches. An attempt will also be made to measure the corresponding aerelastic damping or damping ratio and time required for a number of cycles to elapse for 50% reductions of amplitudes of vibration in a period of an un-damped oscillation of a building.

### SOME RELEVANT LOCAL AND INTERNATIONAL CODE PROVISIONS ON WIND LOADS ESTIMATION

The first step that must be taken in order to effectively and efficiently design a building that is subjected to dynamic wind loading is to correctly obtain or assess the basic wind speed local to the site where the structure is to be constructed.

To correctly assess this, according to Asante-Nimako (1988), the methods for the estimation of wind loads are expressed in various codes of practice in terms of experimental, empirical and analytical approaches. The history of wind loads and its estimation dates back 400 years, when Newton estimates the resistance of a sphere in water and air by swinging at the end of a pendulum with the assumption that the decrease in oscillation
amplitude is due the fluid resistance. This led Newton to postulate the empirical relationship that:

$$\text{Wind load} = CV^2$$  \hspace{1cm} (1)

Where C is an empirical constant and V the maximum sphere velocity in any swing. It is on this empirical relationship that the estimation of wind loads on the structure in general is based today. The practice in the design of structures in general is to design the structure to sustain apart from superimposed dead loads and or superimposed live loads, the loads that may be due to wind action on the structure. The design wind speed is generally based on the results of the statistical analysis of the meteorological records of wind speeds for locations. This depends on sum factors, which will dictate the final value for use in the wind load estimates.

Examining the local situation in Nigeria, Okulaja (1968) carried out relevant studies of extreme wind data for Ikeja/Lagos wind gusts; while Williams (1970) carried out relevant studies involving the evaluation of 50 years data for Nigeria, as part of his estimation of wind loads on engineering structures in Nigeria. Soboyejo (1971) using Gumbel (1954) Type I distribution of yearly highest gust for some meteorological stations in Nigeria came out with Isopleths (Lines of equal wind speed) maps for Nigeria and tables for 2, 5, 10, 25, 50, 75 and 100 year mean recurrent intervals. This effort has been buttressed by the data recently (2002) made available by Nigerian Metrological Agency.

According to Asante-Nimako (1988) the estimation of wind loads is contained in British Standard institution (BSI), code of practice CP3: Chapter V: Part 2 which he remarked as more comprehensive than the Nigerian code of practice NCP1 Part 3 which he described as though followed the BS1 code of practice CP3 approach, is not comprehensive and flexible enough. The local most recently upgraded materials for wind load assessment are BS6399 (2005) and Onundi (2010).

In recent years, wind loads specified in codes and standards have been refined significantly. This is because our knowledge of how the wind affects buildings and structures has expanded due to new technology and advanced research that have ensued in greater accuracy in predicting wind loads. We now have an opportunity to design buildings that will satisfy anticipated loads without excessive conservatism.

According to Kijewski and Kareem (2001), many aspects involved in the estimation of wind loads are held in common by the various codes and standards. First, all the standards break the terrain of any given site down into 3 to 5 categories which will affect the wind characteristics at that location. The design wind speed, associated with one or a range of mean recurrence intervals, used in analysis by each of the codes is typically the product of the basic wind speed and factors to account for the geographic location, topographical effects, and surface roughness, etc. wind gustiness introduces dynamic load effects which the codes and standards account for by factoring up the mean loads by a gust factor. Both time and spatial averaging play an important role in the development of gust factors. For a very small size structure, a short duration gust, which completely engulfs the structure, for example, a 3 s gust may be adequate to account for the effects of gustiness, in which case the gust factor is unity. On the other hand, if the wind-averaging interval is higher, for example, 10 min or more, the averaged wind exhibits less fluctuation, and accordingly the gust factor is greater than unity. This departure from unity is affected not only by the averaging interval, but also by the site terrain and the size and dynamic characteristics of the structure. While all of the standards reference their wind speed at 10 m above ground in a flat, open exposure, each uses gusts of different duration. The British and Canadian standards use the mean hourly wind speed in design, while the European pre standard, the China National Standard and the AIJ Recommendations all use a 10 min mean wind velocity. The ASCE7-95 standard references a 3 s gust, as does the Australian Standard, though, in the latter case. This wind is later converted to a mean hourly wind for subsequent calculations of dynamic pressure and the gust factor. As a result, for any adequate comparison amongst standards, there must be proper adjustments of the reference velocity.

The fundamental characteristic parameters that affect multi-storey building

The fundamental characteristic period, the coefficient of characteristic mode of vibration and the normalized coefficient of characteristic mode of vibration or mode shape factor significantly influence the dynamic loading and perception criteria for the design of tall structures. Therefore, these have been broadly considered by many author and various codes for design as follows (Kijewski and Kareem 1998):

1) The Chinese code mode shape factor of vibration \( \varphi_{sf} \), for buildings, may be taken as:

$$\varphi_{sf} = \tan \left( \frac{\pi}{4} \frac{z}{H} \right)^{0.7}$$

2) The Euro code uses

$$\varphi_{sf} = \left( \frac{z}{H} \right)^{1.5}$$

Where \( z \) and \( H \) are the storey levels and total height of the multi-storey building.
3) Martel (2002) also concluded that significant frequency for design of large engineering structures varies and can be summarized as follows:

a) The maximum accelerations (forces) is commonly at 2 to 10 Hz (T=0.1 to 0.5 s).
b) The maximum velocities (kinetic energy) at 0.5 to 2 Hz (T=0.5 to 2 s).
c) The maximum displacements at 0.006 to 0.5 Hz (T=2 to160 s).

Therefore, designers should avoid structures that are sensitive to wave lengths where wind vortex shedding or seismic forces/energy is concentrated (that is, avoid structures with natural periods that matches the harmful ranges highlighted) depending on whether excessive acceleration, velocity or displacement is of interest for prevention.

According to Martel (2002) in Mexico City, the high rise buildings that were the most damaged as a result of violent dynamic loading were 10 to 30 storeys (30 to 90 m tall) high-rise buildings. These were calculated to have resonant period of 1 to 3 s. Real buildings are sensitive to several different wave frequencies, but will tend to be most sensitive to one frequency (minimum frequency, corresponding to first characteristic mode of frequency):

1) In complying with these recommendations, the works of the following authors are very relevant: Armer and Garas (1982) worked on the adjustment of finite element model for platforms and reported an initial frequency of the first mode of vibration as 0.424 Hz (period T=2.36 s) and measured value of 0.857 Hz (period T=1.167 s) and corrected values as 0.660 Hz (period T=1.515 s).

2) According Armer and Garas (1982), their work on the "accuracy of mathematical models of structural dynamics shows that, theoretically compared results estimates of natural frequencies with the measured and published for 17 buildings and use 163 sample buildings to derive a best fit formula. The comparison of the theory with practice showed that very simple formulae gave estimates of natural frequency that were better correlated with measured natural frequencies than estimates from computer based methods. He recommended the best fit as:

\[ f = 46/H \]

Where H is the total height of the building in meters. This corresponds with a period T = 1.565 s for a 72 m building (that is, T = 0.02174 H).

3) In contrast, Nakashima et al. (1992) as reported by Dowling et al. (1992); in their work "simple expression for predicting fundamental natural periods for high rise building"; surveyed the fundamental natural periods of a total of 239 high-rise steel buildings designed and built in Japan between 1968 and 1988. They confirmed that, all the periods were obtained by eigen value analysis, in which the building was modelled as a discrete spring – mass system with the weight of storey lumped as one discrete mass and the storey stiffness representing one elastic spring. They compared these results with measured values and in particular with the relationship between the fundamental natural period and base shear coefficient and concluded that the fundamental natural period can be successfully computed from 0.026 H; where H is the total height of the building. This complies with a storey top drift angle of 1/200 for a base shear coefficient of 1/(3 T).

### AEROELASTIC MODELLING

According to Kijewski and Kareem (2001), the determination of wind-induced loads and response discussed in their previous works did not account for aeroelastic effects, which can sometimes have significant contributions to the structural response. Response deformations can alter the aerodynamic forces, thus setting up an interaction between the elastic response and aerodynamic forces commonly referred to as aerelasticity. Aeroelastic contributions to the overall aerodynamic loading are distinguished from the unsteady loads by recognizing that aerelastic loads vanish when there is no structural motion. Different types of aeroelastic effects are commonly distinguished from each other. They include vortex-induced vibration, galloping, flutter and aerodynamic damping, respectively.

According to Mendis et al. (2007), the shedding of vortices generates a periodic variation in the pressure over the surface of the structure. When the frequency of this variation approaches one of the natural frequencies of a structure, vortex-induced vibration can occur. The magnitudes of these vibrations are governed both by the structure's inherent damping characteristics ξ and the mass ratio between the structure and the fluid it displaces. These are often combined in the Scruton number defined as:

\[ S_c = \frac{4\pi \xi m}{\rho B^2} \]

Where m is the mass per unit length of the structure and ρ is the density of the fluid (that is, wind) it displaces.

Vortex induced vibration occurs over a range of velocities that increases as the structural damping decreases. Galloping occurs for structures of certain cross-sections at frequencies below those of vortex-induced vibration. Stability of aeroelastic interactions is of crucial importance. The attenuation of structural oscillations by both structural and aeroelastic damping characterizes stability. In an unstable scenario, the motion-induced loading is further reinforced by the body motion, possibly leading to catastrophic failure. Such unstable interactions involve extraction of energy from the fluid flow such that aerodynamic effects cancel structural damping. Flutter is the term given to this unstable situation, which is a common design issue in long span bridges (Mendis et al., 2007).

According to Kijewski and Kareem (2001), depending on the phase of the force with respect to motion, self-excited forces can be associated with the displacement, the velocity or acceleration of the structure. Due to the influence of these associations, these forces can be thought of as "aerodynamic contributions" to stiffness, damping and mass, respectively. Whenever the combined...
Aeroelastic action on various modes results in negative damping for a given mode, will flutter occur? By means of structural dynamics considerations and aerodynamic tailoring flutter must be avoided for the wind velocity range of interest. Even without resulting in flutter, aeroelastic effects can have a significant effect on response.

An illustrative example

A prototype model of 20 storeys building ($H_p=72$ m, $B_p=13$ m and $W_p=25$ m) was tested in an Eiffel-Type boundary layer wind tunnel using a physical model of $H_m=240$ mm, $W_m=83.3$ mm, $B_m=43.3$ mm constructed of a deformable Afara Wood (that is, rectangular and hollow mounted on steel base and Polystyrene - Plate 1 and Figure 1). The prototype frequency $f_p=0.534$ Hz ($f_m=53.4$ Hz), Prototype – model building scale 1 : 300 and Prototype – model wind scale 1 : 3. Basic wind speed of 52.5 m/s was considered for Bauchi - Nigeria. Bauchi is located at an elevation of approximately 610 m above sea level. An average deflection of $\delta_m=19.68$ mm (that is, $\delta_p=93.283$ mm) was measured at the top of the physical model in the wind tunnel.

For the solution to the illustrative example; Since $H > 50$, $H/B=5.54 > 5$ and $0.534 < 1.0$, the structure requires a dynamic analysis (Taranath, 2005; Onundi et al., 2010). Plate 1 is a deformable hollow rectangular Afara wooden physical model and Plate 2 shows the mechanical dial gauge mounted on the physical model which was tested in a subsonic Eiffel type boundary layer wind tunnel (Figure 1) at the Mechanical Technology Department of the Bayero University, Kano-Nigeria. The Afara wood was used for the construction of the superstructure. The Afara which is relatively easy to machine to desired shapes and sizes has been used for the construction of the physical model to ensure that a low mass that satisfies the Scruton number is produced.

The low mass of the model is necessary to ensure that the natural frequency of the model-balance system is well above any expected wind forcing frequency. A primary advantage of this approach is that modal force spectra are obtained directly and can be used in subsequent analytical estimations of building response.
As long as the structural geometry does not change, the forces can be used to analyze the effects of internal structural design changes without the need for further wind tunnel tests (Zhou and Kareem, 2003; Joseph and William, 2006).

The mild steel was used for the base plate which represents an infinitely rigid foundation resting on the polystyrene which is assumed to behave elastic foundation (soil). Its impact on the structural integrity of the model also influences the degree of damping incorporated within the system.

An experimental determination of the model parameters for non-stationary components

From the governing equations of the Dimensional analysis, the solution to the illustrative example is given as:

\[ \frac{\bar{f}}{u} = 0.1323 \] (Reduced frequency or Strouhal number, St)

\[ \text{L}_o = 1.0, \text{ for stationary component and 3.780, for the non-stationary component of the forces} \]

The measured deflection was 19.68 mm from the physical model. Therefore, using the principle of simple proportion and similar triangles with a factor of 1.58 (Simiu and Scanlan, 1996) the corresponding prototype deflection was computed from \( \delta_{p_{\text{max}}} = 0.0158 \eta \delta_{\text{max}} = 0.0158 \times 300 \times 19.68 = 93.2832 \text{ mm} \) or using the tunnel – site wind scale \( \delta_{p_{\text{max}}} = 1.58 \eta \delta_{\text{max}} = 1.58 \times 3 \times 19.68 = 93.2832 \text{ mm} \). \( \eta = \frac{H_p}{H_m} \) - is the Prototype - Physical model length scale while \( \eta = \frac{U_p}{U_m} \) - Site – tunnel's wind scale

The prototype displacement \( \delta_p = 93.283 \text{ mm} \) (converted from the result of the physical model)

\[ \lambda = 1 - \frac{\log L_o}{\log \delta} \]

\[ \eta = 2 + \frac{\log L_o}{\log \delta} \]

\[ : \lambda = 0.70683 \] (that is, for Prototype and Physical model, respectively)

\[ \eta, \gamma = 1, \eta = 2 - \lambda, + \gamma = 2.2932 \]

This result of \( \eta = 2.2932 \), assisted in the determination of the normalized coefficient of first characteristic mode of vibration is given by:

\[ \chi_{ij} = \left( \frac{z}{H} \right)^{\eta} \]

(7)

This quantity is defined as the level fractional contribution to the total structural vibration at storey level \( i \) and corresponding height \( j \) caused by unit acceleration. Whereas the first characteristic mode of vibration is derived by replacing the \( \eta \) with its natural logarithm which is represented by \( m = 0.82995 \):

\[ \chi_{ij}' = \left( \frac{z}{H} \right)^{0.82995} \]

(8)

An experimental determination of number of cycles and damping ratio

According to Thomson (1993), the number of cycles that elapse in 50% reduction in amplitude is given by relationship between amplitude ratios for two consecutive amplitudes is given by Equation 9

\[ \frac{X_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3} = \ldots \ldots \ldots = \frac{X_{n-1}}{X_n} = e^\psi \]

(9)

From the data obtained in the wind tunnel using a mechanical dial gauge, the logarithmic decrement (\( \Psi \)) is defined as:

\[ \psi = \frac{\lambda}{2\pi} \]

(10)
Equation 9 can be rewritten as:

\[
\frac{X_o}{X_n} = \left( \frac{X_o}{X_1} \right) \left( \frac{X_1}{X_2} \right) \left( \frac{X_2}{X_3} \right) \ldots \ldots \ldots = \frac{X_{n-1}}{X_n} = (e^\psi)^{n} = e^{n\psi}
\]

(11)

Where

\[X_o = \text{Amplitude of the first cycle and } X_n = \text{amplitude of the last cycle}\]

Therefore,

\[\psi = \frac{1}{n} \ln \frac{X_o}{X_n}
\]

(12)

From the measured quantity \( \lambda = 0.70683 \), the logarithmic decrement, \( \psi \) is given by:

\[\psi = \frac{2\pi\xi}{\sqrt{1 - \xi^2}} = \frac{\lambda}{2\pi} = 0.1125
\]

(13)

Damping ratio (\( \xi \)) = 0.0179 (for the Prototype and Physical models)

The equation that determines the number of cycles that elapse in 50% reduction in amplitude is given:

\[\psi = \frac{1}{n} \ln \frac{X_o}{X_n} = \frac{1}{n} \ln 2 = \frac{0.693}{n}
\]

(14)

\[n\xi = \frac{0.693}{2\pi} = 0.1103
\]

(15)

According to Thomson (1993), Equation 15 is the equation of rectangular hyperbola. Therefore, number of cycles in a period is:

\[n = \frac{0.693}{0.1125} = 6.162 \approx 7 \text{cycles}
\]

The time for the seven cycles = \( 7T_D = 7 \times 1.8728 = 13.11 \text{ s} \) (that is, for Prototype and Physical model)

The corresponding damping period of oscillation \( (T_D) \) is given by Equation 16.

\[T_D = \frac{2\pi}{\omega_D}
\]

(16)

Since \( \omega_D \) is the damped frequency of the oscillations, \( \omega = 3.3564s^{-1} \) is the model un-damped frequency of the oscillations (Nakashima et al., 1992) and damping ratio (\( \xi \)) = 0.0179

\[\omega_D = \omega \sqrt{1 - \xi^2} = 0.9998\omega = 3.3557\text{sec}^{-1}
\]

(17)

\[T_D = 1.8728 \text{ s} \] (Nakashima et al., 1992)

The un-damped natural frequency for the prototype is give by Equation 18:

\[\omega = \sqrt{\frac{K}{m}}
\]

(18)

Onundi et al. (2010), the value of generalized stiffness \( k' \), analyzed as infinite degree of freedom for multi-storey building is also derived as:

\[m_t = 506.88 \text{ kN/m} = 51.67 \text{ kgs/mm}\]

\[k = 5710.22 \text{ kN/m} = 582.08 \text{ kgs/mm}\]

A critically damped \( (C_{cr}) \) multi-storey building

\[C_{cr} = 2\sqrt{k m} = 346.85 \frac{\text{kg sec}}{\text{mm}}
\]

(19)

Absolute damping (\( C_a \))

\[C_a = \xi C_{cr} = 2.07 \frac{\text{kg s}}{\text{mm}}
\]

Since \( C_{cr} = 115.62 > 6.21 \frac{\text{kg s}}{\text{mm}} \)

The system is under damped (Mario, 1991)

EXPERIMENTAL RESULTS AND DISCUSSION

Figures 2, 3, 4, 5 and 6 have individually and collectively shown that damping ratio is obviously influenced by dependent variables such as stiffness, amplitudes of vibration, \( H/B \) ratio, Strouhal and \( L_n \) numbers under the influence of a basic wind speed of 52.5 ms\(^{-1}\).

Figures 2 and 3 also show that the relationship in the variations of the values of \( H/B \) ratio and relative displacement \( (L_n \text{ number}) \) are inversely proportional to the values of the Strouhal numbers \( (fB/U) \) and damping ratio \( (\xi) \) along the model height. In Figures 4 and 5, as expected, amplitude \( (A_m) \) of the vibrations increased as the model height increased; whereas, a critically damped model \( (C_{cr}) \) and its stiffness \( k \), decrease with increase in model height.

Figures 6, 7 and 8 show progressive decrease in the Strouhal number and damping ratio with increase in model height between 30 and 190 m but a sudden change in gradient of their values occurred at \( H=200 \text{ m} \) where \( H/B = 5.0 \). The reason for this change is clearly explained by Figure 8 which shows the variation of damping ratio as a function of model height and height-breadth ratios \( (H/B) \). While the model height-breadth increased from \( H=30 \text{ with a corresponding } H/B=3.0 \) to \( H=190 \text{ with } H/B=10.56 \), damping ratio \( (\xi) \) decrease from 0.021 to 0.016. Whereas, at a height of \( H=200 \text{ m, } H/B=5.0, \) the damping ratio \( (\xi) = 0.02 \) (Zhou et al., 2003). This result has proved that \( H/B \) ratio seriously influences the values of Strouhal number and damping ratio with increase in model height. Figure 9 show the time required for the numbers of cycles to elapse for 50% reductions of amplitudes of vibration. These times vary between 4.23
Figure 2. Structural parameters for dynamic design of multi-storey building.

Figure 3. Structural parameters for design of multi-storey building.

to 33.98 s for the range of model heights considered under the action of the 52.5 m/s basic wind speed at the local site (Bauchi - Nigeria).

A high rise building with height $H=200$ m, $H/B=5.0$ had the damping ratio ($\xi$) = 0.02 for a composite structural system. This shows that the results of the investigation conducted in the wind tunnel and analyzed with dimensional analysis (DA) is within the acceptable limit. The high rise building...
with height $H=200$ m, $H/B = 5.0$ and damping ratio ($\xi$) = 0.02, is 20% higher than similar system with height $H = 190$ m, $H/B = 10.56$ (110% greater than 5.0) having its damping ratio ($\xi$) = 0.01.

The Council for Tall Buildings and Urban Habitat (CTBUH, 2008) in their recommendations for the seismic design of tall buildings shows that the intrinsic damping of a very tall building, prior to the onset of yielding, will approach that of structural framing alone. Therefore, a damping ratio of between 1 and 2% (that is, 0.01 to 0.02) is recommended as a reasonable range for buildings more than 50 m and less than 250 m in height. The dotted lines in Figures 6, 7 and 8 show that the comparisons of results of the study are within the recommended zone (CTBUH, 2008). Figure 6 particularly shows that a Strouhal numbers of between 0.2 to 0.25...
are most critical for the dynamic study of multi-storey buildings within this range. This recommendation and comparisons are other indications that the values obtained in the study compare favourably with those of other international researchers.

The Indian Code (IS 1893-2002), reversed in 2011; recommended an empirical approximate expression for fundamental natural period of vibration, $T_D = 0.085 H^{0.75}$ (s) for moment resistant framed building without brick infill panel. This gives 2.1 s for the 72 m high building considered in this study. When compared with the 1.873 s used for this result, the former is 12.1% greater than the latter (result of the study) but their recommendation is the criteria for earthquakes resistant design of tall structures,
whereas the latter corresponds with \( T_D = 0.026 \) H recommended by Nakashima et al., (1992) for wind effects on tall structures. Apart from this, the minimum fundamental natural period of vibration is usually sought for design of tall structures because of its significant influence on the fundamental characteristics mode vibration of the high rise buildings.

Another point worthy of note is the influence of the physically measured displacement (at the model top) in a wind tunnel on the experimental determination of the characteristic parameters for dynamic design of multi-storey building subjected to aerodynamic loadings. Apart from the afore-mentioned the investigation revealed that “The product of the Strouhal number, \( \frac{fB}{U} \) and the along model relative displacement \( \left( \frac{\delta}{L_0} \right) \) is equal to the Bernoulli universal constant 0.5. All these are likely to provide wider areas of applicability in structural analysis and in particular experimental use of wind tunnel for structural modeling and similitude in future.

### Conclusion

Although the wind tunnel and dimensional analysis were used in the past and in different ways by many Scholars to investigate the problem of aerodynamic loadings on buildings, such solutions did not include the influence of the building deflection \( \delta \). In this case, the measured deflection has influenced the determination of the measured quantity \( \lambda \) and the logarithmic decrement that led to the computation of other structural parameters that in turn has influence the structural dynamism of the whole system.

The study shows that depending on the phase of the force with respect to motion, self-excited forces can be associated with the displacement, velocity or acceleration of the structure. Due to the influence of these associations, these forces can be thought of as “aerodynamic contributions” to stiffness, damping and mass respectively. Therefore, the building deflection is considered to be an important structural quantity which affects structural phenomena such as forces, moments, velocity and acceleration which severely affect the human perception criteria of occupants using multi-storey buildings. The damping ratio also proved to have been influenced by dependent variables such as stiffness, amplitudes of vibration, \( H/B \) ratio, Strouhal and \( L_0 \) numbers at a basic wind speed of 52.5 ms\(^{-1}\).

This study shows that \( H/B \) ratio seriously influences the values of Strouhal number and damping ratio with increase in model height. It can also be inferred that the relationship in the variations of the values of \( H/B \) ratio and \( L_0 \) number are inversely proportional to the values of the Strouhal numbers \( \left( \frac{fB}{U} \right) \) and damping ratio (\( \xi \)) along the model height. Whereas, the amplitude of the vibrations \( A_m \) progressively increased, while, critically damped model \( (C_{cr}) \) and its stiffness decrease with increase in model height. The high rise building with height \( H = 200 \) m, \( H/B = 5.0 \) and damping ratio (\( \xi \)) = 0.02, is 20% higher than similar system with height \( H = 190 \) m, \( H/B = 10.56 \) (110% greater than 5.0) having its damping ratio (\( \xi \)) = 0.016. Other international researchers recommended a damping ratio of between 1 and 2% (that is, 0.01 to 0.02) for buildings more than 50 m and less than 250 m in height. This recommendation is another indication that the values obtained in the study compares favourably with the conventional norms for high rise buildings.

Another point worthy of note is the influence of the physically measured displacement within a wind tunnel on the experimental determination of the characteristic parameters for dynamic design of multi-storey building subjected to aerodynamic loadings. Apart from the aforementioned the investigation revealed that “The product of the Strouhal number, \( \frac{fB}{U} \) and the along model relative displacement \( \frac{\delta}{\delta^*} = L_0 \) number is equal to the Bernoulli universal constant 0.5. All these are likely to provide wider areas of applicability in structural analysis and in particular, experimental use of wind tunnel for structural modeling and similitude.

### Nomenclatures of symbols

- \( A_m \): Amplitude of the vibrations which increase with the model height
- \( B = D \): Building total width (minimum plan dimension)
- \( B_p \): Building prototype model total width (minimum plan dimension)
- \( B_m \): Building physical model total width (minimum plan dimension)
- \( C \): An empirical constant for assessment of wind loadings
- \( C_a \): Absolutely damping prototype multi-storey building
- \( C_{cr} \): Critically damped prototype multi-storey building
- \( \delta_{m} \): Average physical model deflection
- \( \delta_{p} \): Average prototype model deflection
- \( fB/U \): Strouhal number
- \( f \): Measured natural frequency
- \( f_p \): The prototype model frequency
- \( f_m \): The physical model frequency
- \( H \): The total height of the building in meters
- \( H_p \): Building prototype model total height
- \( H_m \): Building physical model total height
- \( k' \): The value of the generalized stiffness, analyzed as infinite degree of freedom for multi-storey building
- \( L_0 \): \( \frac{\delta}{\delta^*} \): Relative displacement of a damped and undamped model
- \( m \): The mass per unit length of the structure
- \( m' = \ln \eta \)
- \( n \): The number of cycles in a period as defined by the equation of rectangular hyperbola
- \( T \): Natural period of vibration corresponding to first
characteristic mode of vibration (minimum frequency)

\[ U_p \] - Average site wind speed

\[ U_n \] - Average tunnel wind speed

\[ V = U \] - The maximum sphere velocity in any swing for the empirical relationship for the for assessment of wind loadings postulated by Newton

\[ \psi_{sf} \] - Code mode shape factor for vibration

\[ \eta \] - Dynamic coefficient that converts the static relationship between the displacement, and model height

\( H \) to a harmonic function. It varies exponentially with height and time

\[ \eta_h = \frac{H_0}{H_m} \] - Prototype – Physical model length scale

\[ \eta_u = \frac{U_p}{U_n} \] - Site – tunnel’s wind scale

\[ \lambda \] - A coefficient that influences the damping ratio of the model. It is \( 2\pi \) greater than logarithm decrement

\[ \xi \] - The magnitudes of the vibrations governed both by the structure’s inherent damping characteristics

\[ \psi \] - The logarithmic decrement

\[ \rho \] - The density of the fluid (i.e. wind) it displaces

\[ \text{Sc} \] - The Scruton number relates the structure’s inherent damping characteristics \( \xi \) and the mass ratio between the structure and the fluid it displaces

\[ W_B \] - Building prototype model total Breadth (maximum plan dimension)

\[ W_m \] - Building physical model total Breadth (maximum plan dimension)

\[ \omega_d \] - The damped natural frequency of the oscillations

\[ \omega \] - The un-damped natural frequency of the oscillations

\[ x_n \] - Amplitude of the first cycle

\[ x_n \] - Amplitude of the last cycle

\[ x_{ij} \] - Normalized coefficient of first characteristic mode of vibration

\[ \chi_{ij} \] - The first characteristic mode of vibration

\[ z \] - Variation in model height

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