Review

Fundamentals and literature review of wavelet transform in power quality issues

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The effects of power quality (PQ) disturbances on power utilities and power customers are increasing day by day with widespread use of power electronics devices. These disturbances should be detected rapid and accurate. Wavelet transform (WT) has become a popular method to detect, classify and analyze various types of PQ disturbances in power systems. In this study, the fundamentals of WT, its distinctions from Fourier transform and the results of an extensive literature review of published studies are presented. The results of the literature review reveal that the detection of PQ disturbances using WT is a very powerful method and has been used in PQ mitigation devices effectively (that is, Custom Power, Flexible AC Transmission System, Flexible, Reliable and Intelligent Energy Delivery System).

Key words: Wavelet transform, power quality disturbances, literature review, feature extraction.

INTRODUCTION

The term wavelet means a small wave. In addition, a wave refers the condition that this function is an oscillatory. The other term of Wavelet transform (WT) is the mother wavelet. It implies that the functions used in the decomposition and reconstruction processes are derived from one main function or the wavelet. In other words, the mother wavelet is a prototype for generating the other wavelet functions. The term translation (r) is used in the same sense as it was used in short time Fourier transform (STFT). It is related to the location of the window, as the window is shifted through the signal. It is visualized in Figure 1 (Amara, 1995; Robi, 2012).

The application of WT has become a popular for some time under the various names of multirate-sampling (Quadrature Mirror Filters) QMF in electrical engineering (Sarkar et al., 1998). WT possesses many desirable properties that are useful in engineering, economics and finance. The ability of wavelet analysis deals with both the stationary and non-stationary data, their localization in time and the decomposing and analyzing the punctuation in variable signal. While a review of the possible future contributions of WT is provided to the engineering discipline, there are explored two ways in which wavelets might be used to enhance the empirical toolkit of our profession in engineering (Crowley, 2005). These are summarized in the following:

Time scale versus frequency: An examination of data sets to assess the presence and ebb and the flow of frequency components is potentially valuable (Crowley, 2005).

Time scale decomposition: Recognition components of the signal can be possibly found at ‘disaggregate’ (scale)

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level rather than an aggregate level forecasting: disaggregate forecasting, establishing global versus local aspects of series (Crowley, 2005).

In this study, the basic principles of WT especially Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT), the properties of most used Haar and Daubechies wavelets are presented. The similarity and contrast points of WT with the Fourier Transform (FT) are also given in this study. The recent studies with the subject of WT applications in power quality (PQ) issues are examined in details in the last part of the work.

CONTINUOUS WAVELET TRANSFORM

Unlike FT, CWT possesses the ability to construct a time-frequency representation of a signal that offers very good time and frequency localization. In other words, CWT is used to divide a continuous-time function into the equal-time intervals and equal-frequency intervals. In CWT, the signal is multiplied with the wavelets, same as STFT and the transform is computed separately for different parts of the time-domain signal. The mathematical definition of CWT is given in the following (Wikipedia, 2012).

$$\psi(\tau,s) = \frac{1}{\sqrt{|s|}} \int x(t) \psi^* \left( \frac{t-\tau}{s} \right) dt$$  \hspace{1cm} (1)

The scale ($s$) and $\tau$ parameters are the parameters of CWT functions. The function $\psi(\tau,s)$ named as mother wavelet is the transforming function. Let, $x(t)$ be the signal to be analyzed. The mother wavelet is chosen to serve as a prototype for all windows in the process. All the windows used for the dilated, compressed and shifted are the versions of the mother wavelet. There are many functions used for this purpose. Once the mother wavelet is chosen in CWT, the computation can be started from scale $s=1$ and it is continued with increasing the values of $s$. In this situation, the analyzing of the signal is started from the low frequencies and proceeded towards the high frequencies. This procedure is repeated for every value of $s$. Every computation for a given value of $s$ fills the corresponding single row of the time-scale plane (Robi, 2012). This process is summarized in Figure 2.

An accurate signal analysis is the proper analysis of
each trace. FT is not a good tool for this purpose. It can only provide frequency information of the oscillations that comprise the signal. It gives no direct information about when an oscillation is occurred. Another tool for signal analysis is STFT. The full-time interval is divided into a number of small, equal-time intervals. These are individually analyzed by using FT. The result contains time and frequency information. However, there is a problem with this approach. The equal-time intervals are not adjustable. When the time intervals are very-short-duration, high-frequency bursts are hard to detect (Sarkar, 1998; Morlet, 1982a, b).

CWT is a good tool for analysis the signal included the harmonics both the time-domain and the frequency-domain. The signal, that its frequency is changed with time, is detected easily when the time intervals are very-short-duration. The most common wavelets are summarized in the following.

Haar Wavelet

There are two functions that play a primary role in the wavelet analysis. These are the scaling function ($\phi$) and wavelet ($\psi$). These functions generate a family of functions that can be used to break up or reconstruct the signal. To emphasize the marriage involved in building this family, $\phi$ is sometimes called as father wavelet and $\psi$ as mother wavelet (Boggess, 2001). The simplest wavelet analysis is based on the Haar scaling function illustrated in Figure 3.

The Haar scaling function is defined as

$$\phi(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$  \hspace{1cm} (2)

The function $\phi(x-k)$ is obtained by shifted versions of $\phi(x)$ to the right by $k$ units (assuming $k$ is positive). Let, $V_0$ be the space of functions in the form of $\phi(x-k)$. It is given in the following.

$$\sum_{k \in \mathbb{Z}} a_k \phi(x-k)$$  \hspace{1cm} (3)

where $k$ can range over any finite set of positive or negative integers.

The thinner blocks are required to analyze the high-frequency signals. The function $\phi(2x)$, whose width is half of $\phi(x)$, is illustrated in Figure 4.

The simplest $\psi(x)$ of Haar wavelet is the function that consists of two blocks and it can be written as (4).

$\psi(x) = \phi(2x) - \phi(2(x-1/2))$  \hspace{1cm} (4)

Haar wavelets have one particularly desirable property that they are zero everywhere except on a small interval. If the function is closed everywhere outside of a bounded interval, it has compact support. If a function has compact support, the smallest closed interval on which the function has non-zero values is called the support of the function (Aboufadel et al., 1999). If $k \neq 0$ and $\psi(x)$ is orthogonal to $\phi(x)$, then the compact support of $\psi(x)$ and the compact support of $\phi(x-k)$ do not overlap in the Cartesian coordinate system.

So, $\int_{-\infty}^{\infty} \psi(x)\phi(x-k)dx = 0$. Consequently, $\psi(x)$ belongs to $V_1$ and it is orthogonal to $V_0$. Then, $\psi(x)$ is called Haar wavelet. It is given in Figure 5.

In general, $n^{th}$ generation of daughters will have $2^n$ wavelets and it is defined as (5).

$$\psi_{n,k}(t) = \psi(2^n t-k), \hspace{1cm} 0 \leq k \leq 2^n - 1$$  \hspace{1cm} (5)
Daubechies Wavelet

Ingrid Daubechies in 1987 sought a wavelet family that had compact support and some sort of smoothness. The scaling function of her discovery was quite a feat and was greeted with enthusiasm. There are three requirements for the following example of Daubechies wavelets.

(i) The scaling function has compact support that \( \phi(x) \) is zero outside of the interval 0\(<\)t<3.
(ii) The orthogonality condition is the heart of any multiresolution analysis.
(iii) The regularity condition is related with the smoothness of the scaling function.

The dilation equation of Daubechies wavelet is given in the following.

\[
\phi(t) = \frac{1+\sqrt{3}}{4} \phi(2t) + \frac{3+\sqrt{3}}{4} \phi(2t-1)
\]

\[
+ \frac{3-\sqrt{3}}{4} \phi(2t-1) + \frac{1-\sqrt{3}}{4} \phi(2t-1)
\]

Daubechies wavelets have the compact support, orthogonality and regularity conditions. These properties are to be required in constructing the scaling function, if the accuracy is desired. Hence, the mother wavelet and all children can be approximated as well. This indicates that it is not specifically necessary to know a formula for the scaling function in order to work with wavelets. Other wavelet families are developed in this way (Aboufadel et al., 1999; Addison, 2002).

DISCRETE WAVELET TRANSFORM

DWT is the form of CWT that uses the discrete values of the signal in the time-domain. The mathematical expression of DWT is given in the following.

\[
\psi_{m,n}(t) = \frac{1}{\sqrt{a_0^m}} \left( \frac{t-nb_0}{a_0^m} \right)
\]

where the integers \( m \) and \( n \) control the wavelet dilation and translation respectively, \( a_0 \) is a specified fixed dilation step parameter which should be greater than one and \( b_0 \) is the location parameter which should be greater than zero (Addison, 2002).

The control parameters \( m \) and \( n \) contain the set of positive and negative integers. WT of a continuous signal \( x(t) \) gives the decomposition wavelets by using the discrete form of (1). In other words, the scale-location of the signal in the time-frequency-domain is determined by using (8).

\[
T_{m,n} = \int_{-\infty}^{\infty} x(t) a_0^{-m/2} \psi \left( a_0^{-m} t - nb_0 \right) dt
\]

where the values of \( T \) are the scale-location grid of the signal in the time-frequency-domain according to indexes \( m \) and \( n \).

For DWT, the values of \( T_{m,n} \) are known as wavelet coefficients or detail coefficients. Common choices for DWT parameters, \( a_0 \) and \( b_0 \) are selected as two and one, respectively. The term "power of two" is emerged for the practical application. This power of two is known as the dyadic grid arrangement for dilation and translation steps. The dyadic grid is perhaps the simplest and most efficient discretization for practical purposes and lends itself to the construction of an orthonormal wavelet basis (Aboufadel et al., 1999; Addison, 2002).

\[
\psi_{m,n}(t) = 2^{-m/2} \psi \left( 2^{-m} t - n \right)
\]

By choosing an orthonormal wavelet basis \( \psi_{m,n}(t) \), the original signal can be reconstructed with utilizing the wavelet coefficients \( T_{m,n} \) by using inverse discrete wavelet transform (IDWT) given in the following.

\[
x(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} T_{m,n} \psi_{m,n}(t)
\]

ADVANTAGES AND DISADVANTAGES OF WT WITH FFT

Many transforms have been devised by engineers and mathematicians such as the Laplace transform, the Hilbert transform, Z-transform and FT. However, FT is the most popular among these transforms. It presents lots of
limitations in many areas of study. Only one of them is present at a time. The frequency-domain gives no information about the time of signal generation or the time at which the frequencies are occurred (Bousaleh et al., 2009). The points of similarity and contrast between WT and FT are given in the following (Saribulut, 2012).

(i) The matrix of inverse transform is the transpose of the original for both FFT and DWT. Both transforms can be viewed as a rotation in a function space to a different domain. For FFT, this new domain contains basis functions included in the sinus and cosine functions. For WT, this new domain contains more complicated basis functions called wavelets, mother wavelets or analyzing wavelets (Amara, 1995; Sarkar et al., 1998).

(ii) The basis functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and scalegrams useful at picking out frequencies and calculating power distributions (Amara, 1995).

(iii) The most interesting dissimilarity between these two kinds of transforms is that the individual wavelet functions are space whereas localized. However, the sine and cosine functions of FT are not space whereas localized. This localization feature makes many functions and operators along with wavelets’ localization of frequency when it is transformed into the wavelet domain (Amara, 1995).

(iv) An advantage of WTs is that the windows vary. In order to isolate signal discontinuities, some very short basis functions would be required. At the same time, some very long basis functions would be required in order to obtain the detailed frequency analysis (Sarkar et al., 1998).

LITERATURE REVIEW OF WAVELET TRANSFORM IN POWER QUALITY ISSUES

The latest studies related with WT are summarized as literature survey in the following:

A conventional synchronous fundamental DQ-frame algorithm is modified in filtering process (Firouzjah et al., 2009). As an improvement, the Fourier-based low-pass filter is replaced with a windowing-wavelet method. The adopted windowing aspect is Hamming window to reduce conventional rectangular window effect in the frequency-domain. The disturbance classification schema is performed with a wavelet-neural network (Uyar et al., 2008). It performs feature extraction and classification algorithm composed of a wavelet feature extractor based on norm entropy and classifier based on a multi-layer perceptron.

A transmission-line fault location model based on an Elman recurrent network has been presented for balanced and unbalanced short circuit faults (Ekici et al., 2009). WT is used for selecting distinctive features about the faulty signals. The fault signals are analyzed by using DWT which splits the signal into detail and approximation coefficients. The energies of detail coefficients fed neural networks to estimate fault location. The recognition of PQ disturbances is studied (Kaewars et al., 2008). The multi-wavelet-based neural classifier is used to automatically detect, localize and classify the transient disturbance type for high accuracy and low usage time. PQDs classification based on WT and self-organizing learning array system is proposed (He et al., 2006). WT is utilized to extract feature vectors for various PQ disturbances based on the multi-resolution analysis. Then, these feature vectors are applied to a solar system for training and testing.

An integrated approach for the PQ data compression by using the Spline WT and ANN is presented (Meher et al., 2004). Its performance is assessed in terms of compression ratio, mean square error and percentage of energy retained in the reconstructed signals. A compression technique for power disturbance data via DWT and the wavelet packet transform is introduced (Hamid et al., 2004). The compression technique is performed through the signal decomposition up to a certain level, threshold of wavelet coefficients and signal reconstruction. A fuzzy-wavelet packet transform based PQ indices are introduced (Morsi et al., 2009). Online real-time detection and classification of voltage events in power systems are presented (Pérez et al., 2008). Wavelet analysis is used for detection and estimation of the time-related parameters of an event and the extended Kalman’s filtering is used for confirmation of the event and for computation of the voltage magnitude during the vent.

An algorithm based on the wavelet-packet transform for the analysis of harmonics in power systems is given (Barros et al., 2008). The proposed algorithm decomposes the voltage/current waveforms into the uniform frequency bands. It uses a method to reduce the spectral leakage due to the imperfect frequency response of the used wavelet filter bank. The use of WT and the computational intelligence techniques are presented to quantify voltage short-duration variation in electric power systems (Machado et al., 2009). WT is used to determine the event duration and to obtain the characteristic curve that is related with the signal norm as the function of the number of cycles for a waveform without disturbance.

A fault location procedure for distribution networks based on the wavelet analysis of the fault-generated traveling waves is presented (Borghetti et al., 2008). CWT is applied to the voltage waveforms recorded during the fault in correspondence of a network bus. An automatic classification of different PQD by using WT and fuzzy k-nearest neighbor based classifier is proposed (Panigrahi et al., 2009). The training data samples are generated using parametric models of the PQ disturbances. The features are extracted using some of
the statistical measures on the WT coefficients of the disturbance signal when decomposed up to the fourth level. A method for detection and classification of PQ disturbances is proposed (Masoum et al., 2010). The distorted waveforms (PQ events) are generated based on the IEEE 1159 standard, captured with a sampling rate of 20 kHz and de-noised using DWT to obtain signals with the higher signal-to-noise ratio.

An implementation of DWT using LC passive filters for operating a DVR system is presented (Saleh et al., 2008). The proposed implementation is based on designing Butterworth LC passive filters with cutoff frequencies that are identical to cutoff frequencies of DWT associated digital filters. An integrated model for recognizing PQDs using a novel wavelet multiclass-support vector machine is proposed (Lin et al., 2008). Various disturbance events were tested for the proposed method and the wavelet-based multilayer-perceptron neural network was used for comparison. To enhance the performance of WT-based monitoring systems and to improve the classification accuracy of WT based classifiers, two standards statistical procedures have been proposed (Dwivedi et al., 2009). PQ waveform data has been performed by the correct implementation of wavelet threshold using the supreme functional of the Brownian bridge process.

A non-invasive methodology to evaluate and classify the electrical system failures is presented (Sartori et al., 2010). It is based on the electrical system magnetic signature recognition by using the wavelet signal decomposition and the variance spectrum evaluation, respectively. A simple and effective technique to de-noise PQ waveform data for enhanced detection and time localization of PQ disturbances is proposed by using inter and inter-scale dependencies of wavelet coefficients (Dwivedi et al., 2010). The proposed method operates on WT coefficients that are obtained from wavelet multiresolution decomposition of PQ waveform data and exploits the local structure of wavelet coefficients within a sub-band as well as across the scales to produce noise-free coefficients.

A new algorithm based on WT to detect the starting point, ending point and the magnitude of the voltage sag is developed (Gencer et al., 2010). DWT is used to detect fast changes in the voltage signals which allow time localization of differences frequency components of a signal with different frequency wavelets. WT is used to reformulate the recommended PQ levels (Morsi et al., 2010). The non-stationary waveforms are analyzed for the smart grid. An effective technique is proposed by using inter and inter-scale dependencies of wavelet coefficients to de-noise the waveform of PQ data for enhanced detection and time localization of PQ disturbances (Dwivedi et al., 2010).

An algorithm for continuous real-time power waveforms capture, compression, and transmission is presented (Tse et al., 2012). The integer-lifting WT with integer mapping and an adaptive threshold scheme are used for truncating wavelet coefficients and compressing the non-stationary component, respectively. Automatic classifications of PQ events and disturbances by using WT and support vector machine (SVM) are presented (Erkst et al., 2012). WT is used in order to obtain the distinctive features of event signals. PQ event types are determined by using SVM classifier. At the last stage of intelligent recognition system, types of PQ disturbances regarding each fault event are identified by doing a further analysis. An approach for the recognition and classification of PQ disturbances by using WT and wavelet based SVM is presented (Moravej et al., 2010). The proposed method employs WT techniques to extract the most important and significant feature from details and approximation waves. The obtained severable feature vectors are used for training SVMs to classify the PQ disturbances.

The use of near perfect reconstruction filter banks on the harmonic analysis of power system waveforms is investigated (Taskovski et al., 2012). A dual-purpose wavelet-based transient detection module to perform the fault detection and the voltage regulation through conditional threshold selection are described (Lakshmi et al., 2013). An effective boundary element numerical algorithm combining with WT and generalized minimum residual method for the solution of the electric field problem in substations are proposed (Deng et al., 2012). A wavelet-based methodology for the characterization of voltage sags is described (Costa et al., 2013). Frequency-Shifting wavelet decomposition via the Hilbert transform is introduced for PQ analysis introduced (Tse et al., 2012). The proposed algorithm overcomes the spectra leakage problem in DWT. A hybrid model is proposed for monthly runoff prediction by using wavelet transform and feed forward neural networks (Okkan, 2012). DWT and Levenberg-Marquardt optimization algorithm based feed forward neural networks are considered for the modeling study. The two hybrid AI-based models which are reliable in capturing the periodicity features of the process are introduced for watershed rainfall-runoff modeling (Nourani et al., 2011). In the first model, an ANN is used to find the non-linear relationship among the residuals of the fitted linear SARIMAX model. In the second model, wavelet transform is linked to the ANFIS concept and the main time series of two variables are decomposed into some multi-frequency time series by wavelet transform.

CONCLUSIONS

Power quality disturbances might have large financial impacts on the customers of modern power industrial. New power electronics based devices can be used to mitigate these PQ disturbances. These disturbances in power systems should be detected, classified and analyzed with certain accuracy. The number of studies
related with the applications of Wavelet Transform in these devices has been increasing. WT performs a superior performance in the applications of power systems. It extracts the distinct features of typical PQ disturbances to obtain dynamic parameters superior to traditional methods. A comprehensive review of WT in PQ issues has also been carried out to explore a broad perspective on their applications. The literature review reveals more precise in discriminating the type of transients, higher robustness and faster processing time for detecting and classifying voltage and current disturbing events.

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